WEEKLY TEST TYJ TEST - 13 Balliwala
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[PHYSICS]

1.
2.
3. In return journey the angle made by the projectile with the horizontal is $-\theta$. Hence,

$$\vec{\Delta p} = [\hat{i} p\cos\theta + \hat{j} p\sin\theta]$$

$$-[\hat{i} p\cos(-\theta) + \hat{j} p\sin(-\theta)] = \hat{j}\, 2p\sin\theta$$

or $|\vec{\Delta P}| = 2p\sin\theta$

4. $$\frac{h_2}{h_1} = \frac{u^2\sin^2\theta_2}{2g} \times \frac{2g}{u^2\sin^2\theta_1}$$

$$= \frac{\sin^2\theta_2}{\sin^2\theta_1} = \frac{\sin^2\pi/6}{\sin^2\pi/3} = \frac{1}{3}$$

$\therefore$ $h_2 = h_1/3$

5. Time taken by bullet to reach the target $= \frac{\text{distance}}{\text{velocity}} = \frac{\text{distance}}{u\cos\theta}$

As θ is very small, $\cos\theta = 1$

$$\text{Time} = \frac{\text{distance}}{u} = \frac{400}{400} = 1\,\text{sec}$$

Vertical deflection of bullet

$$= \frac{1}{2}gt^2 = \frac{1}{2}\times 10 \times (1)^2 = 5\text{ metre}$$

6. $R_{max.} = \frac{u^2}{g} = 1000\text{ m}$ (R is maximum where $\theta = 45^o$)

$$H = \frac{u^2\sin^2\theta}{2g} = \frac{u^2}{2g}\times\sin^2 45^o = \frac{u^2}{4g}$$

$$= \frac{1000}{4} = 250\text{ m}$$

7. Kinetic energy is minimum at the higest point and highest point is attained after covering distance equal to 0.5 R
8. When the horizontal range is maximum, the maximum height attained is R/4. Hence, co-ordinates of the point = (400, 100).
9. R is same for both θ and $(90^o - \theta)$,
If angle w.r.t. vertical is 40^o, then w.r.t. horizontal direction in will be $90^o - 40^o = 50^o$.
10. $$\frac{R}{T^2} = \frac{u^2\sin 2\theta \times g^2}{g.4u^2\sin^2\theta} = \frac{g}{2}\cot\theta$$

i.e., $gT^2 = 2R\tan\theta$
If T is doubled, then R becomes 4 times

11. The shooter has to direct slightly upward as the path followed by a projectile is a parabola trajectory.

12. $H_{max.} = \frac{u^2 \sin^2 \theta}{2g}$

$T = \frac{2u\sin\theta}{g}$

$\frac{H_{max.}}{T^2} = \frac{u^2 \sin\theta}{2g} \times \frac{g^2}{4u^2 \sin^2 \theta}$

$= \frac{g}{8} = \frac{10}{8} = \frac{5}{4}$

13. $H = \frac{u^2 \sin^2 \theta}{2g}$

or $80 = \frac{u^2 \sin^2 \theta}{2 \times 10}$

or $u^2 \sin^2\theta = 1600$

or $u \sin\theta = 40\ ms^{-1}$.

Horizontal velocity = $u \cos\theta$ = at

$= 3 \times 30 = 90\ ms^{-1}$

$\frac{u\sin\theta}{u\cos\theta} = \frac{40}{90}$

or $\tan\theta = \frac{4}{9}$ or $\theta = \tan^{-1}\left(\frac{4}{9}\right)$

14. $\theta_1 = 30^o$, $\theta_2 = 60^o$

$H = \frac{u^2 \sin^2 \theta}{2g}$, i.e., $H \propto \sin^2\theta$

$\therefore \frac{H_1}{H_2} = \frac{\sin^2 \theta_1}{\sin^2 \theta_2} = \frac{\sin^2 30^o}{\sin^2 60^o}$

$= \frac{(1/2)^2}{(\sqrt{3}/2)^2} = \frac{1}{3}$

15. Component of velocity $\perp$ to plane remains the same (in opposite direction),

i.e., $u \sin\theta = 20 \sin 30^o = 10$ m/s.

[MATHEMATICS]

31. (c) Given, diameter of circular wire $= 10cm$, therefore length of wire $= 10\pi$.

Hence required angle $= \frac{\text{arc}}{\text{radius}} = \frac{10\pi}{50} = \frac{\pi}{5}$ *radian*.

32. (d) Given expression is

$\sin^2 5^o + \sin^2 10^o + \sin^2 15^o + ... + \sin^2 85^o + \sin^2 90^o$.

We know that $\sin 90^o = 1$ or $\sin^2 90^o = 1$.

Similarly, $\sin 45^o = \frac{1}{\sqrt{2}}$ or $\sin^2 45^o = \frac{1}{2}$ and the angles are in A.P. of 18 terms. We also know that

$\sin^2 85^o = [\sin(90^o - 5^o)]^2 = \cos^2 5^o$.

Therefore from the complementary rule, we find $\sin^2 5^o + \sin^2 85^o = \sin^2 5^o + \cos^2 5^o = 1$.

Therefore,

$\sin^2 5^o + \sin^2 10^o + \sin^2 15^o + ... + \sin^2 85^o + \sin^2 90^o$

$= (1+1+1+1+1+1+1+1) + 1 + \frac{1}{2} = 9\frac{1}{2}$.

33. (b) We have $x+\frac{1}{x}=2\cos\theta$,

Now $x^3+\frac{1}{x^3}=\left(x+\frac{1}{x}\right)^3-3x\frac{1}{x}\left(x+\frac{1}{x}\right)$

$=(2\cos\theta)^3-3(2\cos\theta)=8\cos^3\theta-6\cos\theta$

$=2(4\cos^3\theta-3\cos\theta)=2\cos 3\theta$.

Trick : Put $x=1 \Rightarrow \theta=0^\circ$.

Then $x^3+\frac{1}{x^3}=2=2\cos 3\theta$.

34. (a) $\sin x+\operatorname{cosec} x=2 \Rightarrow (\sin x-1)^2=0 \Rightarrow \sin x=1$

$\therefore \sin^n x+\operatorname{cosec}^n x=1+1=2$.

35. (b) Since $\cos^6\alpha+\sin^6\alpha+K\sin^2 2\alpha=1$ using formula $a^3+b^3=(a+b)^3-3ab(a+b)$ and on solving, we get the required result *i.e.* $K=\frac{3}{4}$.

36. (c) $\sin 20^\circ \sin 40^o \sin 60^\circ \sin 80^\circ$ $=\frac{1}{2}\sin 20^\circ \sin 60^\circ (2\sin 40^o \sin 80^\circ)$

$=\frac{1}{2}\sin 20^\circ \sin 60^\circ(\cos 40^\circ-\cos 120^\circ)$

$=\frac{1}{2}.\frac{\sqrt{3}}{2}\sin 20^\circ\left(1-2\sin^2 20^\circ+\frac{1}{2}\right)$

$=\frac{\sqrt{3}}{4}\sin 20^\circ\left(\frac{3}{2}-2\sin^2 20^\circ\right)$

$=\frac{\sqrt{3}}{8}(3\sin 20^\circ-4\sin^3 20^\circ)$ $=\frac{\sqrt{3}}{8}\sin 60^\circ=\frac{\sqrt{3}}{8}.\frac{\sqrt{3}}{2}=\frac{3}{16}$.

37. (b) $x=\sin 130^o\cos 80^o,\ y=\sin 80^o\cos 130^o$

$\Rightarrow x=\cos 40^o\cos 80^o,\ y=-\sin 80^o\sin 40^o$

So, $x>0$ and $y<0$ and $xy<0$

Now, $z=1+xy \Rightarrow 0<z<1$.

38. (a, c) We have $\sin\alpha=1/\sqrt{5} \Rightarrow \cos\alpha=2/\sqrt{5}$

and $\sin\beta=3/5 \Rightarrow \cos\beta=4/5$

$\sin(\beta-\alpha)=\sin\beta\cos\alpha-\sin\alpha\cos\beta$

$=\frac{3}{5}.\frac{2}{\sqrt{5}}-\frac{1}{\sqrt{5}}.\frac{4}{5}=\frac{2}{5\sqrt{5}}=0.1789$

Now $\sin\frac{\pi}{4}=\frac{1}{\sqrt{2}}=0.7071=\sin\frac{3\pi}{4}$

Since $0<0.1789<0.7071$

$\therefore \sin 0<\sin(\beta-\alpha)<\sin\frac{\pi}{4} \Rightarrow 0<(\beta-\alpha)<\frac{\pi}{4}$

Also, $\sin\pi<\sin(\beta-\alpha)<\sin\frac{3\pi}{4}$

$\therefore (\beta-\alpha)\in[0,\pi/4]$ and $[3\pi/4,\pi]$.

39. (b) Let $f(x) = \sin\theta + \cos\theta = \sqrt{2}\sin\left(\theta + \frac{\pi}{4}\right)$

But $-1 \le \sin\left(\theta + \frac{\pi}{2}\right) \le 1 \Rightarrow -\sqrt{2} \le \sqrt{2}\sin\left(\theta + \frac{\pi}{4}\right) \le \sqrt{2}$.

Hence the maximum value of $(\sin\theta + \cos\theta)$

i.e., of $\sqrt{2}\sin\left(\theta + \frac{\pi}{4}\right) = \sqrt{2}$.

$\therefore \sin\left(\theta + \frac{\pi}{4}\right) = 1 \Rightarrow \sin\left(\theta + \frac{\pi}{4}\right) = \sin\frac{\pi}{2}$

$\Rightarrow \theta + \frac{\pi}{4} = \frac{\pi}{2} \Rightarrow \theta = \frac{\pi}{4} = 45^o$.

40. (b) $5 - 5\sin^2\theta + 7\sin^2\theta = 6 \Rightarrow 2\sin^2\theta = 1$

$\Rightarrow \sin^2\theta = \frac{1}{2} = \sin^2\left(\frac{\pi}{4}\right) \Rightarrow \theta = n\pi \pm \frac{\pi}{4}$.

41. (b) $\tan\theta = \frac{-1}{\sqrt{3}} = \tan\left(\pi - \frac{\pi}{6}\right)$, $\sin\theta = \frac{1}{2} = \sin\left(\pi - \frac{\pi}{6}\right)$ and $\cos\theta = \frac{-\sqrt{3}}{2} = \cos\left(\pi - \frac{\pi}{6}\right)$

Hence principal value is $\theta = \frac{5\pi}{6}$.

42. (a) $2\sin 3x\cos x - 2\sin 3x = 0$, $\therefore \sin 3x = 0$, $\cos x = 1$

$\Rightarrow 3x = n\pi$ or $x = \frac{n\pi}{3}$ and $x = 2n\pi$

The second value $x = 2n\pi$ is included in the value given by $x = \frac{n\pi}{3}$.

43. (c) $(5 + 4\cos\theta)(2\cos\theta + 1) = 0$

$\cos\theta = -5/4$, which is not possible.

$\therefore 2\cos\theta + 1 = 0$ or $\cos\theta = -1/2$

$\Rightarrow \theta = \frac{2\pi}{3}, \frac{4\pi}{3}$. Solution set is $\left\{\frac{2\pi}{3}, \frac{4\pi}{3}\right\} \in [0, 2\pi]$.

44. (a) $\tan 2\theta = \cot\theta \Rightarrow \tan 2\theta = \tan\left(\frac{\pi}{2} - \theta\right)$

$\Rightarrow 2\theta = n\pi + \frac{\pi}{2} - \theta \Rightarrow \theta = \frac{n\pi}{3} + \frac{\pi}{6}$.

45. (d) $\frac{1 - \cos 2\theta}{1 + \cos 2\theta} = 3 \Rightarrow \frac{1 - (1 - 2\sin^2\theta)}{1 + (2\cos^2\theta - 1)} = 3$